Key 66 pts.

3.12 Double Angle Identities

Name____________________ **Date** __________ **Period** __________

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Use the appropriate sum or difference identity to prove the double-angle identity. Show work!

3 1) $\cos 2u = \cos^2 u - \sin^2 u$ 1) ________

$\cos(u+u) = \cos u \cdot \cos u - \sin u \cdot \sin u$

$= \cos^2 u - \sin^2 u$ ✓

3 2) $\cos 2u = 1 - 2\sin^2 u$ 2) ________

$\cos(u+u) = \cos u \cdot \cos u - \sin u \sin u$

$= \cos^2 u - \sin^2 u$

$= (1 - \sin^2 u) - \sin^2 u = 1 - 2\sin^2 u$ ✓

Find all solutions to the equation in the interval [0, 2π). Show all work!

4 3) $\sin 2x = 2 \sin x$ 3) $x = 0, \pi$

$\sin 2x - 2\sin x = 0$

$2\sin x \cos x - 2\sin x = 0$

$2\sin x(\cos x - 1) = 0$

$2\sin x = 0$ $\quad$ $\cos x - 1 = 0$

$\sin x = 0$ $\quad$ $\cos x = 1$

$x = 0, \pi$ $\quad$ $x = 0$

5 4) $\cos 2x = \sin x$ 4) $x = \pi/6, 5\pi/6$ & $3\pi/2$

$\cos 2x - \sin x = 0$

$(1 - 2\sin^2 x) - \sin x = 0$

$2\sin^2 x + \sin x - 1 = 0$

Factor

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ $\quad$ $\sin x + 1 = 0$

$\sin x = 1/2$ $\quad$ $\sin x = -1$

$x = \pi/6$ & $5\pi/6$ $\quad$ $x = 3\pi/2$

8 5) $\sin 2x - \tan x = 0$ 5) $x = 0, \pi, \pi/4,$ ~~$3\pi/4, 5\pi/4$~~ $, 7\pi/4$

$\frac{\cos x}{\cos x}\left(\frac{2\sin x \cos x}{1}\right) - \frac{\sin x}{\cos x} = 0$

$\frac{2\sin x \cos^2 x - \sin x}{\cos x} = 0$

$\cos x \rightarrow \cos x \neq 0 \quad x \neq \pi/2, 3\pi/2$

$2\sin x \cos^2 x - \sin x = 0$

$\sin x(2\cos^2 x - 1) = 0$

$\sin x = 0$ $\quad$ $\cos^2 x = 1/2$

$x = 0, \pi$ $\quad$ $\cos x = \pm\sqrt{2}/2$

$x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

Rewrite with only sin x and cos x. Show all work!

2 6) $\sin 2x + \cos x$ 6) ________

$(2\sin x \cos x) + \cos x$ or $\cos x(2\sin x + 1)$

2 7) sin 2x + cos 2x 7) ____________

$2\sin x\cos x + 2\cos^2 x - 1$ or $2\sin x\cos x + 1 - 2\sin^2 x$

or $2\sin x\cos x + \cos^2 x - \sin^2 x$

4 8) sin 2x + cos 3x = 8) ____________

possible answers after simplifying:

1) $2\sin x\cos x + 4\cos^3 x - 3\cos x$

2) $2\sin x\cos x + \cos^3 x - 3\sin^2 x\cos x$

3) $2\sin x\cos x + \cos x - 4\sin^2 x\cos x$

$= 2\sin x\cos x + \cos(2x+x)$

$= 2\sin x\cos x + \cos 2x \cdot \cos x - \sin 2x\sin x$

(use identity here (show work)

Prove the identity. Show all work!

2 9) sin 4u = 2 sin 2u cos 2u 9) ____________

rewrite

$\sin(2(2u)) = 2\sin 2u\cos 2u$ ✓

or $\sin(2u+2u) = 2\sin 2u\cos 2u$ ✓

3 10) 2 csc 2x = csc² x tan x → right side $\frac{1}{\sin^2 x}\cdot\frac{\sin x}{\cos x} = \frac{\sin x}{\sin^2 x\cos x} = \frac{1}{\sin x\cos x}$ 10) ____________

left side $\frac{2}{\sin 2x}$

$= \frac{2}{2\sin x\cos x} = \frac{1}{\sin x\cos x}$

same ✓

3 11) sin 3u = (sin u)(4 cos² u – 1) 11) ____________

$\sin(2u+u) = \sin 2u\cos u + \sin u\cos 2u$

$= (2\sin u\cos u)\cos u + \sin u(2\cos^2 u - 1)$

$= \sin u(2\cos^2 u + 2\cos^2 u - 1) = \sin u(4\cos^2 u - 1)$ ✓

4 12) cos 4u = 1 – 8 sin² u cos² u 12) ____________

$\cos(2u+2u) = \cos 2u\cos 2u - \sin 2u\sin 2u$

$= (2\cos^2 u - 1)(1 - 2\sin^2 u) - (2\sin u\cos u)(2\sin u\cos u)$

$= 2\cos^2 u - 4\cos^2 u\sin^2 u - 1 + 2\sin^2 u - 4\sin^2 u\cos^2 u$

$= 2\cos^2 u + 2\sin^2 u - 1 - 8\sin^2 u\cos^2 u$

$= 2(\cos^2 u + \sin^2 u) - 1 - 8\sin^2 u\cos^2 u$

$= 1 - 8\sin^2 u\cos^2 u$ ✓

Solve algebraically for exact solutions in the interval [0, 2π). Show all work!

13) $\cos 2x + \cos x = 0$ 13) ________

5

$2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = 1/2 \quad \cos x = -1$

$x = \pi/3$ & $5\pi/3$

& $x = \pi$

14) $\cos x + \cos 3x = 0$

8

$\cos x + \cos(2x + x) = 0$

$\cos x + \cos 2x \cos x - \sin 2x \sin x = 0$

$\cos x + (2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x = 0$

$\cancel{\cos x} + 2\cos^3 x - \cancel{\cos x} - 2\sin^2 x \cos x = 0$

$2\cos x(\cos^2 x - \sin^2 x) = 0$

$2\cos x = 0$

$x = \pi/2$ & $3\pi/2$

$\cos x = \sin x$

$x = \pi/4, 3\pi/4, \frac{5\pi}{4}, 7\pi/4$

15) $\sin 2x + \sin 4x = 0$ 15) ________

10

$\sin 2x + 2\sin 2x \cos 2x = 0$

$\sin 2x(1 + 2\cos 2x) = 0$

$\sin 2x = 0$

$2\sin x \cos x = 0$

$\sin x = 0 \quad \cos x = 0$

$x = 0, \pi \quad x = \pi/2$ & $3\pi/2$

$1 + 2\cos 2x = 0$

$1 + 2(2\cos^2 x - 1) = 0$

$1 + 4\cos^2 x - 2 = 0$

$4\cos^2 x - 1 = 0$

$\cos x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$x = \pi/3, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

16) BONUS: $\sin 2x - \cos 3x = 0$ 16) ________

Have fun!